

# Unbundling and Incumbent Investment in Quality Upgrades and Cost Reduction\*

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## Abstract

We study the investment of a telecommunications incumbent in quality and in cost reduction when an entrant can use its network through unbundling of the local loop. We find that unbundling may lower incentives for quality improvements, but raises incentives for cost reduction. Therefore, it is not true that all types of investment are crowded out with unbundling. If the regulator can commit to a socially optimal unbundling price before investment, the incumbent makes both types of investment. In the absence of commitment, the incumbent will not invest, so that unbundling regulation may lower welfare as compared to no regulation.

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# 1 Introduction

In the last 10 years, mandatory unbundling has become a standard remedy proposal for solving the bottleneck problem in fixed telecoms competition. Since there are high entry barriers in the telecommunications market, because of scale economies, sunk costs and first-mover advantages, it is hard for a new operator to enter the market as a full-facility competitor. In particular, the building of local access networks, which are composed of circuits connecting end users to switches located in central offices, requires large investments in terms of money and time.

Under mandatory unbundling an incumbent firm has to share the use of some of its facilities with its competitors. This implies that an essential input is, at the wholesale level, separated from the incumbent's overall facilities, in order to allow for commercial wholesale supply of this input. In the particular case of local loop unbundling, this means that a new operator can directly plug into the incumbent's network by creating a connection from its switch to the incumbent's local access network (Figure 1). This policy is supposed to generate entry in the market, and to encourage entrants to build their own network in the future when their stock of costumers is large enough.

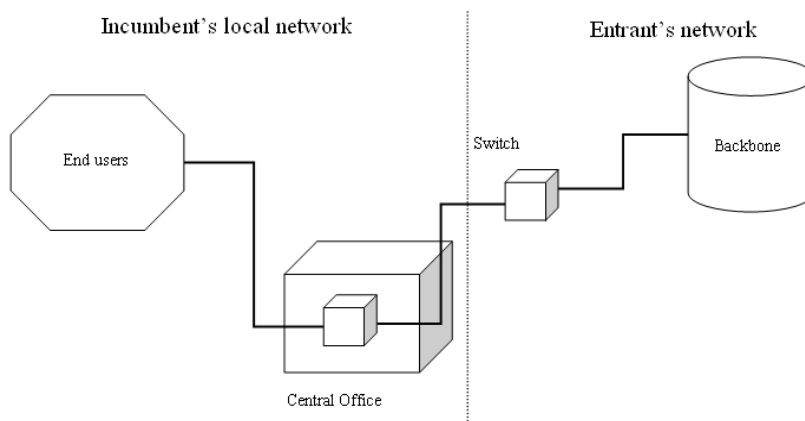


Figure 1

Mandatory unbundling is promoted both in the United States and the European Union. In the US, the Telecommunications Act of 1996 requires that incumbents unbundle their networks.<sup>1</sup> Incumbents and entrants are then required to negotiate a price for the entrant's use of an unbundled element.

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<sup>1</sup>In its Local Competition Order in August 1996, the FCC specified seven unbundled elements: local loops, network interface devices, local and tandem switching, interoffice transmission facilities, signaling networks, operations support systems and operator services and assistance.

If they are unable to reach an agreement, the price is determined by the regulator. The calculation of the regulated prices is guided by a framework called Total Element Long Run Incremental Cost (TELRIC). This follows a forward looking methodology based on the assumption that an efficient and modern network is in place.<sup>2</sup>

In the EU, the New Regulatory Framework (NRF) of 2003 embraces the view that regulation should be used to actively promote service-based competition by facilitating access to existing infrastructure.<sup>3</sup> But even before the NRF, European legislation mandated the provision of unbundled access.<sup>4</sup> While recognizing that infrastructure competition is the primary means to attain sustainable competition in telecommunications because it increases the pressure to minimize costs and induces a higher scope for innovation, the European Commission sustains that service competition is a necessary pre-requisite for infrastructure competition. According to the Commission, competition would never be able to develop in the short term if entrants were not able to gain access to the incumbent's network.

Service-based competition promoted by unbundling has been criticized on the basis that it only promotes static efficiency. The main argument is that incumbents would not have incentives to invest if they had to share the benefits of their investments with rivals. Moreover, if access to the incumbents' network was allowed too easily, this would create inefficiencies in the long run since an entrant would not have incentives to build competing facilities (see Jorde et al., 2000).

Partially as a response to these arguments, several empirical studies analyzing the effect of unbundling on incumbent firms' investment have emerged. For instance, Willig et al. (2002) examine the relationship between unbundling prices and Bell companies' investments. They test two opposite hypotheses. The first is the investment deterrence hypothesis, according to which a low unbundling price encourages utilization by the entrants and, as a consequence, the incumbents invest less. The second one is the competitive stimulus hypothesis, according to which a low unbundling price encourage entry, and this increased competition strengthens the incumbents' incentive to invest. Their results support the second hypothesis, and therefore they conclude that lower unbundling prices stimulate incumbents' investment. A

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<sup>2</sup>See more about the Long Run Incremental Cost methodology in Laffont and Tirole (2000).

<sup>3</sup>Directive 2002/19/EC of the European Parliament and of the Council of 7 March 2002 on access to, and interconnection of, electronic communications networks and services.

<sup>4</sup>Regulation 2887/2000 of the European Parliament and of the Council of 18 December 2000 on unbundled access to the local loop.

study by Hassett et al. (2003) obtains similar conclusions.

However, the Willig et al. (2002) result is not without controversy. Haring et al. (2002) criticize Willig's estimation methodology and develop their own econometric model. They obtain the opposite relationship, i.e., low unbundling prices reduce the profitability of incumbents' investment leading to a reduction in that investment. Hausman and Sidak (2005) corroborate this opinion in their case study about the unbundling experience in the US, New Zealand, Canada, United Kingdom, and Germany. Gabel and Huang's (2003) econometric results indicate that in the US the higher the unbundling price, the more likely is the introduction of new services by the incumbents. Ingraham and Sidak (2003) show that mandatory unbundling increases the volatility of the incumbents' stock returns, which increases their equity cost.

In 2003 a new controversy has emerged after the publication of the Phoenix Center Policy Bulletin n°5, which shows that the rise in unbundling lines has increased investment by incumbents. This gave origin to two replies, one by Hazlett et al. (2003) on behalf of Verizon, and another by Hill (2003) on behalf of Z-Tell-Communications, both contesting the empirical estimation and arguing that the rise in unbundling lines has led to a decline in incumbents' investment. As a response, the Phoenix Center published its Policy Bulletin n°6 which, by incorporating the comments of the two replies, shows that its previous result was robust.

Finally, there is also a study by Chang et al. (2003) that finds, using US data, that lower access rates have spurred investment in digital systems by incumbent local carriers. Even so, the same study points in the opposite direction for Europe.

We can conclude that there is an unresolved controversy about the true effects of unbundling prices in incumbent's investments and, following from this, what the regulated unbundling price should be. In this paper we will focus on these two points distinguishing between investment in cost reduction and in quality upgrades.

In contrast with the large amount of research on static access pricing (Armstrong, 2002), the dynamic study of optimal access pricing is still in its early stages. Valletti (2003) reviews the existing literature about the relationship between access pricing and investment, and provides a discussion about investment incentives by relating them with questions common to R&D.

Foros (2004) shows that under some conditions the investment level in quality is lower with price regulation, since the access price is set equal to marginal cost. Kotakorpi (2006) considers a similar model with vertical differentiation, and obtains similar results. Cambini and Valletti (2004) study the impact of access charges on the incentives to invest, but in a context

of two-way access. They derive the result that firms would choose a price above marginal cost in order to diminish each other's incentives to invest. In addition, there are some papers that consider cost-reducing investments, as Biglaiser and Ma (1999), Cabral and Riordan (1989) and Sappington (2002), the first in a context of an incumbent firm and the other two in monopoly. Vareda and Hoernig (2006) study both the incumbent and entrant's investment in the building of a new network.

In our paper, we develop a theoretical model with two operators that offer differentiated services, and try to explain the relationship between the unbundling price and the investment made by the incumbent. Since it is a partial consumer participation model, it portrays non-mature markets, such as the broadband market. Bourreau and Dogan (2005) assume full consumer participation represented by a Hotelling model. In this model profits are insensitive to the unbundling price for a large interval of unbundling prices, which does not seem to be reasonable in the context of investment choice.

The main contribution of our model is the comparison of the incumbent's incentives for two different types of investment: quality-upgrades and cost-reduction. We show that, although these investments are complements, the direct effect of the unbundling price on each one differs. Indeed, a lower unbundling price decreases incentives for quality improvements, but raises incentives for cost reduction. This follows from the fact that, for a lower unbundling price, the incumbent wants to maintain its competitive advantage. Thus, it has more incentives to invest in cost reduction increasing its cost-advantage. On the other hand, it has less incentives to invest in quality upgrades because this benefits both operators. In equilibrium, we always have a higher investment in cost reduction for a lower unbundling price, while investment in quality can be higher or lower due to the complementarity relationship.

We also determine the socially optimal unbundling price. First, we assume that the decision about the unbundling price is taken before investment and that the regulator commits to it until the end of the game. We show that the regulator sets a higher unbundling price when the cost of improving quality is relatively low, in order to give incentives for this type of investment. When cost reduction is less expensive, then the unbundling price the regulator should set is lower. We contrast these results with a context where the regulator cannot commit to his decisions and revises the unbundling price after the investment has been made. In this case, the incumbent does not invest since the regulator sets a price such that it earns negative profits in case of investment. Social welfare is then lower in a no-commitment context.

We compare both contexts with an unregulated market. We show that the incumbent has always incentives to unbundle its infrastructure in order to attract new consumers to the market.<sup>5</sup> This is always worse than the context where the regulator sets the unbundling price before investment as the price set by the incumbent is too high, but it can be better than a no-commitment context since there is some investment. Therefore, we conclude that the unbundling problem raised by some authors is more a problem of commitment rather than unbundling as such.

Finally, we provide a short analysis of the case of mature markets. In these markets, the investment in quality upgrades increases with the unbundling price, while the investment in cost reduction is independent of it. A relevant example is fixed telephony.

The remainder of the paper is organized as follows. We describe the model in Section 2. In Section 3 we obtain the equilibrium prices and quantities, and in Section 4 we find the profit-maximizing investments. Then, in Section 5, we solve the regulator's problem. In Section 6 we find the unregulated market equilibrium and compare it with the regulated contexts, and in Section 7 we analyze a mature market. Finally, in Section 8 we conclude.

## 2 The Model

We introduce a model of a telecommunications market, where two firms compete on subscription prices and supply horizontally differentiated services. The operators on this market are: one vertically integrated network (denoted as incumbent) which owns the local loop, and one non-integrated network (denoted as entrant) which only owns a backbone and switches, and needs access to the incumbent's local loop.

We introduce a third party, a regulator, who sets the unbundling price in order to maximize social welfare. We assume that the unbundling price is the only instrument available for the regulator. This corresponds closely to the current European practice. First, we consider a context where the regulator fixes the unbundling price at the beginning of the game. Later, we consider a context where the regulator only takes the final decision about the unbundling price after the investment stage. We adopt the simplifying assumption of complete information, i.e., the regulator is supposed to have full information about the incumbent's technology and costs.

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<sup>5</sup>Some incumbents have voluntarily entered into agreements with entrants for unbundled access. For example, Verizon and Covad in the US.

We assume that the incumbent can invest in its infrastructure both to increase quality and to reduce cost. We also assume that there is no uncertainty about returns on investment, i.e., for a given amount of expenditures on investment a given effect is obtained for sure.

After observing the price set by the regulator and the investment made by the incumbent, the entrant decides if it asks for access to the local loop. In this paper we exclude the possibility of entering as a facility-based competitor.

### Demand side

In a telecommunications market consumers usually subscribe services from only one operator, thereby, they face a discrete decision problem of which operator to subscribe to. However, if we aggregate the demand of all consumers and divide by their number, we obtain the demand of a representative consumer who subscribes services from both operators. We can then use a quasi-linear consumer surplus function similar to Bowley (1924):

$$U(q_I, q_E) = a_I q_I + a_E q_E - \frac{1}{2} b (q_I^2 + 2\theta q_I q_E + q_E^2) - p_I q_I - p_E q_E, \quad (1)$$

where  $a_i$  is the reservation price for service  $i$  and  $\theta$  indicates the degree of substitutability. When  $\theta$  is higher the services are stronger substitutes. In the extreme case, when  $\theta = 0$  we have independent services, and when  $\theta = 1$  we have perfect substitutes.  $(q_I, p_I)$  is the number of subscribers and the subscription price of the incumbent, while  $(q_E, p_E)$  is the number of subscribers and the subscription price of the entrant. For simplicity, we assume that  $b = 1$ .

If we solve the representative consumer's problem:

$$\max_{q_I, q_E} U(q_I, q_E), \quad (2)$$

we obtain the following demand functions:

$$q_I = \frac{a_I - \theta a_E - p_I + \theta p_E}{1 - \theta^2} \quad (3)$$

$$q_E = \frac{a_E - \theta a_I - p_E + \theta p_I}{1 - \theta^2} \quad (4)$$

(these expressions are valid provided that  $\theta < 1$  and that both  $q_I, q_E \geq 0$ ).

We assume partial consumer participation. Contrary to the Hotelling model often used in literature, consumer participation depends on price, which creates the opportunity to consider welfare effects neglected by it.

### Supply side

Regarding the incumbent's cost structure, we assume that cost per subscriber is just a constant marginal cost  $c$ . For simplicity, irrespective of being the incumbent or the entrant that sells the services to subscribers, we assume the incumbent's marginal cost per subscriber to be the same.

If the entrant decides to ask for access to the incumbent's local loop, the incumbent receives from the entrant a price  $r$  per rented line (unbundling price). For its own retail services it receives a subscription price  $p_I$  per consumer. Given these, in the absence of investment, incumbent's profit is equal to:

$$\pi_I = (p_I - c) q_I + (r - c) q_E. \quad (5)$$

The entrant receives a subscription price  $p_E$  from its customers and pays the correspondent unbundling price to the incumbent. Hence, its profit is:

$$\pi_E = (p_E - r) q_E. \quad (6)$$

The regulator maximizes social welfare, which is the following:

$$W = \pi_I + \pi_E + CS, \quad (7)$$

where  $CS$  is consumer surplus.

Throughout, we make the following assumptions:

*Assumption 1:* Reservation prices are equal for both operators and higher than marginal cost:

$$a_I = a_E = a > c.$$

*Assumption 2:* Firms only operate in the market if they have non-negative profit:

$$\pi_I \geq 0, \pi_E \geq 0.$$

According to Assumption 1, if entry occurs, the entrant is restricted to providing services with a quality equal to the incumbent's. This happens because it depends on the incumbent's infrastructure, thus it is not able to supply services that the incumbent could not supply, too.

As we are not considering in our model questions related with foreclosure, we assume that the entrant has already incurred in a sunk cost of entry. Thus, it asks for access if it is able to obtain non-negative retail profits.

### Investments

As we have said, the incumbent has the possibility to invest in its network. We will consider two types of investment: quality-upgrades and cost reduction.

In the first case, we assume that the investment increases the reservation price by  $g$ , which implies a parallel shift in both demand functions. In fact, as the entrant supplies its services through the incumbent's local network, it also benefits from this investment, consequently, the reservation price for its services also increases in  $g$ . We assume that spillovers are complete, contrary to Foros (2004) and Kotakorpi (2006).<sup>6</sup> An example of this kind of investment is an upgrade of the switching equipment or the installation of new fibre optic cables, which allows to increase the velocity of the transmission or the capacity to deliver voice and data traffic.

The investment cost function is quadratic and given by:

$$C_q(g) = \frac{\alpha}{2}g^2. \quad (8)$$

The second type is an investment to decrease the cost of providing the services by turning the local network more efficient and reliable. Since the entrant uses the incumbent's lines to supply the services to its subscribers, it is the incumbent that supports all the operating costs. Consequently, if the incumbent invests in cost reduction, the marginal cost of supplying all consumers is reduced, no matter whether they are the incumbent's or the entrant's. Since the incumbent has constant marginal cost  $c$ , the innovation represents a decrease of  $h$  in marginal cost.

The investment cost function is also quadratic:

$$C_c(h) = \frac{\beta}{2}h^2. \quad (9)$$

### Timing of the game

1. The regulator fixes the unbundling price.
2. The incumbent decides how much to invest in its infrastructure.
3. The entrant decides if it asks for access.
4. Firms compete in prices.

When the regulator cannot commit to his decision the order of the first two moves is reversed.

We now find the Subgame-Perfect Equilibrium using backward induction.

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<sup>6</sup>In these papers, an investment increases the willingness to pay for services, but the dimension of the effect depends on the ability of each operator to transform input to output. Thus, we can have the incumbent offering higher quality services, and vice-versa.

### 3 Price competition stage

For given reservation prices (and therefore quality levels) and cost levels, using demand functions (3) and (4), and maximizing profit with respect to subscription price, we obtain the following Nash-equilibrium prices and quantities of the price competition stage (see Appendix A):

$$p_I - c - h = \frac{1 - \theta}{2 - \theta} (x_0 + g + h) + \frac{3\theta}{4 - \theta^2} (y_0 + h) \quad (10)$$

$$p_E - r = \frac{1 - \theta}{2 - \theta} (x_0 + g + h) - 2 \frac{1 - \theta^2}{4 - \theta^2} (y_0 + h) \quad (11)$$

$$q_I = \frac{1}{2 - \theta^2 + \theta} (x_0 + g + h) - \frac{\theta}{4 - \theta^2} (y_0 + h) \quad (12)$$

$$q_E = \frac{1}{2 - \theta^2 + \theta} (x_0 + g + h) - \frac{2}{4 - \theta^2} (y_0 + h), \quad (13)$$

where  $x_0 = a - c$  and  $y_0 = r - c$ .

Parameter  $y_0$  represents the incumbent's *ex ante* access margin. It can also be interpreted as the entrant's cost disadvantage, since the entrant has to pay  $r$  for each line while the incumbent only incurs a cost of  $c$ . In the future we will work with  $y_0$  when we want to find the optimal unbundling price.

Firms' profit and welfare become:

$$\begin{aligned} \pi_I = & \frac{1}{(2 - \theta)^2} \left( \frac{1 - \theta}{1 + \theta} (x_0 + g + h)^2 - \frac{8 + \theta^2}{(2 + \theta)^2} (y_0 + h)^2 \right. \\ & \left. + \frac{4 - 2\theta + \theta^2}{2 + \theta} (x_0 + g + h) (y_0 + h) \right) - \frac{\alpha}{2} g^2 - \frac{\beta}{2} h^2. \end{aligned} \quad (14)$$

$$\begin{aligned} \pi_E = & \frac{1 - \theta}{(2 - \theta)^2} \left( \frac{1}{1 + \theta} (x_0 + g + h)^2 + 4 \frac{1 + \theta}{(2 + \theta)^2} (y_0 + h)^2 \right. \\ & \left. - \frac{4}{2 + \theta} (x_0 + g + h) (y_0 + h) \right) \end{aligned} \quad (15)$$

$$\begin{aligned} W = & \frac{1}{(2 - \theta)^2} \left( \frac{3 - 2\theta}{1 + \theta} (x_0 + g + h)^2 - \frac{1}{2} \frac{4 + 5\theta^2}{(2 + \theta)^2} (y_0 + h)^2 \right. \\ & \left. - (1 - \theta) (x_0 + g + h) (y_0 + h) \right) - \frac{\alpha}{2} g^2 - \frac{\beta}{2} h^2. \end{aligned} \quad (16)$$

These functions are only valid if the entrant asks for access.

**Lemma 1** *The entrant only asks for access if:*<sup>7</sup>

$$y_0 \leq \bar{y}_0 \equiv \frac{1}{2} \frac{2 + \theta}{1 + \theta} (x_0 + g + h) - h. \quad (17)$$

**Proof.** Equivalent to  $\pi_E \geq 0$ . ■

Note that when services are more substitutable, i.e., when  $\theta$  is close to 1, the entrant's equilibrium profit tends to zero and the incumbent's equilibrium profit tends to the monopolist's profit at  $p_I = r$ .

In monopoly, profit and welfare become:

$$\pi_I^M = \frac{1}{4} (x_0 + g + h)^2 - \frac{\alpha}{2} g^2 - \frac{\beta}{2} h^2 \quad (18)$$

$$W^M = \frac{1}{2} (x_0 + g + h)^2 - \frac{\alpha}{2} g^2 - \frac{\beta}{2} h^2. \quad (19)$$

## 4 Investment stage

We start to solve the incumbent's problem about how much to invest in quality and in cost reduction when the regulator acts as a first-mover.

**Proposition 2** *The investments in quality upgrades and cost reduction are complements. Moreover, the marginal revenue of investing in quality upgrades (cost reduction) is increasing (decreasing) in the unbundling price.*

**Proof.** From the derivatives of (14) we easily find that:

$$\frac{\partial^2 \pi_I}{\partial g \partial h} > 0, \quad \frac{\partial^2 \pi_I}{\partial g \partial y_0} > 0, \quad \frac{\partial^2 \pi_I}{\partial h \partial y_0} < 0.$$

■

The higher is the investment in quality upgrades, the higher is the marginal benefit of investing in cost reduction, since it decreases the cost of serving a higher number of subscribers. Thus, investments are complements. However, they are affected differently by the unbundling price. When the unbundling price is higher, the incumbent earns more profit with the entrant's subscribers. As a consequence, it has a higher incentive to invest in quality upgrades since this increases the entrant's number of subscribers. On

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<sup>7</sup>This condition is also sufficient to guarantee that  $q_I, q_E \geq 0$  and  $p_E \geq r$  at equilibrium prices.

the other hand, it has less incentives to invest in cost reduction since this decreases its rival's number of subscribers. This follows from the fact that this investment only reduces incumbent's cost per line, while the entrant's cost per line, which is given by  $r$ , remains the same. Consequently, the incumbent's cost advantage over its rival increases. That is why, despite the complementarity between the two investments, we may observe each investment going in a different direction after a change in the unbundling price

If the regulator set an access margin instead of an unbundling price, so that  $r$  decreased when  $c$  decreased, the investment in cost reduction would be equivalent to an investment in quality upgrades, since there would not be any gains in terms of cost advantage after an investment in cost reduction.

Given the unbundling price set by the regulator, the incumbent maximizes its profit function (14) with respect to  $g$  and  $h$ . The profit-maximizing investments in quality upgrades and in cost reduction are:

$$g^* = \frac{(8 + \theta^3)(1 + \theta)\beta - (2 + \theta)(6 - \theta + \theta^2)}{V(\theta, \alpha, \beta)}y_0 \quad (20)$$

$$+ \frac{(6 - \theta + \theta^2)(2 + \theta) + 2(1 - \theta)(2 + \theta)^2\beta}{V(\theta, \alpha, \beta)}x_0$$

$$h^* = -\frac{(1 + \theta)(8 + 2\theta^2 - \theta^3)\alpha - (2 + \theta)(6 - \theta + \theta^2)}{V(\theta, \alpha, \beta)}y_0 \quad (21)$$

$$+ \frac{(2 + \theta)(8 - 3\theta^2 + \theta^3)}{V(\theta, \alpha, \beta)}\alpha x_0,$$

where:

$$V(\theta, \alpha, \beta) = (1 + \theta)(4 - \theta^2)^2\alpha\beta - (2 + \theta)(6 - \theta + \theta^2) \quad (22)$$

$$- 2(1 - \theta)((2 + \theta)^2\beta + (4 + 4\theta - \theta^3)\alpha).$$

*Assumption 3:* The socially optimal quality upgrades and cost reduction investments are finite, which is equivalent to have:<sup>8</sup>

$$\alpha > \underline{\alpha} \equiv \frac{6 - 4\theta}{4 - 3\theta^2 + \theta^3} \quad (23)$$

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<sup>8</sup>These are obtained by solving the problem of a social planner who takes the decisions regarding investments and unbundling price.

$$\beta > \underline{\beta} \equiv \frac{(1 - \theta) (12 + 8\theta - \theta^2 - 2\theta^3) \alpha + (7 + 3\theta + 7\theta^2 + \theta^3)}{(2 + \theta)^2 ((2 - \theta)^2 (1 + \theta) \alpha - (6 - 4\theta))}. \quad (24)$$

If both these conditions are verified we have  $V(\theta, \alpha, \beta) > 0$ , and both  $h^*$  and  $g^*$  are a maxima.<sup>9</sup>

Note that (20) and (21) are the profit-maximizing investments given that  $h^* < c$ . However, if  $c$  is low enough, we may have  $h^* > c$ , and in this case the best the incumbent can do is to invest  $h^{**} = c$ . The profit-maximizing investment in quality upgrades is then:

$$g^{**} = \frac{(1 + \theta) (4 - 2\theta + \theta^2)}{((1 + \theta) (2 - \theta)^2 \alpha - 2(1 - \theta)) (2 + \theta)} (y_0 + c) \quad (25)$$

$$+ \frac{2(1 - \theta)}{(1 + \theta) (2 - \theta)^2 \alpha - 2(1 - \theta)} (x_0 + c)$$

Having found the equilibrium investments in quality upgrades and in cost reduction, we are now able to determine the effect of unbundling on each type:

**Proposition 3** *In the presence of both types of investment and for  $c > h^*$ , the profit-maximizing investment in cost reduction is decreasing in the unbundling price.*

**Proof.** Taking the derivative of  $h^*$  with respect to  $r$ , we obtain:

$$\frac{\partial h^*}{\partial r} = \frac{\partial h^*}{\partial y_0} \frac{\partial y_0}{\partial r} = - \frac{(1 + \theta) (8 + 2\theta^2 - \theta^3) \alpha - (2 + \theta) (6 - \theta + \theta^2)}{V(\theta, \alpha, \beta)} < 0,$$

which is always positive for  $\alpha > \underline{\alpha}$ . ■

**Proposition 4** *Define  $\tilde{\beta} = \frac{6 - \theta + \theta^2}{(1 + \theta)(4 - 2\theta + \theta^2)}$ . In the presence of both types of investment and for  $c > h^*$ , the profit-maximizing investment in quality upgrades is increasing in the unbundling price if  $\beta > \tilde{\beta}$  and decreasing if  $\beta < \tilde{\beta}$ .*

**Proof.** Taking the derivative of  $g^*$  with respect to  $r$ , we obtain:

$$\frac{\partial g^*}{\partial r} = \frac{\partial g^*}{\partial y_0} \frac{\partial y_0}{\partial r} = \frac{(8 + \theta^3) (1 + \theta) \beta - (2 + \theta) (6 - \theta + \theta^2)}{V(\theta, \alpha, \beta)},$$

which is positive for  $\beta > \tilde{\beta}$  and negative for  $\beta < \tilde{\beta}$ . ■

<sup>9</sup>These conditions are sufficient to guarantee that the Hessian of the incumbent's problem verifies the maxima conditions.

As we have seen in Proposition 2, when the unbundling price is lower, the marginal revenue of investing in cost reduction is higher. Thus, as expected, we have a higher equilibrium investment in this type.

In contrast, the effect of the unbundling price in the equilibrium quality improvements does not follow immediately from Proposition 2, as only for a high  $\beta$  we obtain a positive relationship. This results from the complementarity relationship between the two investments. Indeed, if we take into account the indirect effect of a higher unbundling price through cost reduction, we observe that this has a negative impact on the marginal revenue of quality improvements. Consequently, when this indirect effect is relatively higher, we obtain a negative relationship between investment in quality upgrades and the unbundling price. This happens for a low  $\beta$ , i.e., when the reaction of cost reduction to an increase in the unbundling price is high.

The same indirect effect is present in the cost reduction equilibrium investment, but in this case the effect is weaker for all  $\alpha > \underline{\alpha}$ , and consequently, we always obtain a negative relationship. In fact, as the marginal revenue of cost reduction reacts more to changes in  $r$  than the marginal revenue of quality improvements, the direct effect of an increase in the unbundling price on cost reduction is higher.

For  $c < h^*$  cost reduction is independent of the unbundling price, while quality improvements is always increasing in  $r$ , since the indirect effect does not exist in equilibrium. However, for the rest of the paper, we will assume that  $c > h^*$  so that we always have (20) and (21) as the investment choices of the incumbent. In this case, we obtain the following result:

**Proposition 5** *Let  $\Phi = \left(\frac{8+2\theta^2-\theta^3}{8+\theta^3}\right)$ . When  $c > h^*$ , if  $\frac{\beta}{\alpha} > \Phi$ , the profit-maximizing total investment is increasing in the unbundling price and if  $\frac{\beta}{\alpha} < \Phi$ , it is decreasing.*

**Proof.** Summing  $g^*$  and  $h^*$ , and then taking the derivative with respect to  $r$ , we obtain:

$$\frac{\partial (h^* + g^*)}{\partial r} = (1 + \theta) \frac{(8 + \theta^3) \beta - (8 + 2\theta^2 - \theta^3) \alpha}{V(\theta, \alpha, \beta)},$$

and thus  $\frac{\partial (h^* + g^*)}{\partial r} > 0$  if and only if  $\beta > \left(\frac{8+2\theta^2-\theta^3}{8+\theta^3}\right) \alpha$ . ■

According to this Proposition, if cost reduction is sufficiently expensive as compared with quality improvements, the higher is the unbundling price, the higher is the total amount of investment we expect the incumbent to

do. Otherwise, we expect total investment to be lower when the unbundling price is higher.

Note that  $\Phi > 1$  for  $\theta \in (0, 1)$ , i.e., we can have a  $\beta > \alpha$  and even so the relationship is negative. This is a consequence of the stronger direct effect of a higher unbundling price on cost reduction.

The existent empirical studies do not distinguish between these two types of investment. By this way, it is natural that we observe some contradictory results about the relationship between the unbundling price and the incumbent's investment. In fact, if  $\frac{\beta}{\alpha} > \Phi$ , a more intense utilization generated by a lower unbundling price leads an incumbent to invest less. Therefore, we expect to see more investment when the unbundling price is higher, which confirms the results of Haring et al. (2002) and Gabel and Huang (2003). If  $\frac{\beta}{\alpha} < \Phi$ , we obtain a negative relationship between investment and unbundling price, which is according to the result by Willig et al. (2002), Hassett et al. (2003) and the Phoenix Center Studies (2003), which state that a lower unbundling price increases the intensity of competition, and this increases the incentives of an incumbent to invest in order to gain a competitive advantage.

## 5 Regulation stage

### 5.1 A no-investment benchmark

Let us consider first the absence of an investment stage. In this case, a regulator maximizes social welfare over  $r$  without having to take into account the incumbent's investment incentives. Thus, given our assumption that incumbent must make non-negative profits, we obtain the following result:

**Proposition 6** *In the absence of an investment stage, the second-best socially optimal unbundling price is such that the incumbent earns zero profits.*

**Proof.** See Appendix B. ■

The regulator wants the incumbent to subsidize the entrant's activity through a negative access margin. In fact, when the unbundling price is lower than marginal cost, competition between operators is more intense. The incumbent wants the entrant to have fewer subscribers in order to lose less money with unbundled lines, and the entrant wants to have more subscribers because profit per subscriber is higher. As a result, the subscription price of both operators decreases, increasing social welfare.

## 5.2 Commitment to unbundling price before investment

In a commitment context the regulator sets the unbundling price before the incumbent takes its decision about investment and commits to it until the end of the game. Hence, when he decides, the regulator takes into account how the incumbent will invest given the unbundling price. This implies that he has three objectives: He wants to increase the intensity of competition, to give incentives for an investment in cost reduction and to give incentives for an investment in quality upgrades. When  $\beta \leq \tilde{\beta}(\theta)$  the three objectives are all favored by a low  $r$ , and therefore the first-best unbundling price ( $y_0^{c1}$ ) is so low that the incumbent would earn *ex post* negative profits. In this case, the second-best unbundling price ( $y_0^{c2}$ ) set by the regulator is such that the incumbent earns *ex post* zero profits. Only for  $\beta > \tilde{\beta}(\theta)$  quality improvements become increasing in  $r$ , and the regulator has incentives to set a higher  $r$ . However, if  $\beta$  is lower than a threshold  $\bar{\beta}(\theta, \alpha)$ , the first-best unbundling price is still such that the incumbent would earn *ex post* negative profits. Therefore, the regulator continues to set a second-best unbundling price. Only when  $\beta > \bar{\beta}(\theta, \alpha)$  quality improvements become sufficiently important so that the first-best  $r$  allows the incumbent to earn *ex post* positive profits, and thus it can be implemented by the regulator.

We then have the following results:

**Proposition 7** Define  $\bar{\beta}(\theta, \alpha)$  by  $\pi_I^*(y_0^{c1}(\theta, \alpha, \bar{\beta}, x_0), \theta, \alpha, \bar{\beta}, x_0) = 0$ . When the regulator sets the unbundling price before the investment decision:

(a) At the socially optimal the incumbent earns *ex post* positive profits for  $\beta > \bar{\beta}(\theta, \alpha)$ , and *ex post* zero profits for  $\beta \leq \bar{\beta}(\theta, \alpha)$ .

(b) The socially optimal unbundling price is increasing in  $\beta$  and decreasing in  $\alpha$  when  $\beta > \bar{\beta}(\theta, \alpha)$ , and when  $\beta \leq \bar{\beta}(\theta, \alpha)$  it is increasing both in  $\alpha$  and  $\beta$ .

**Proof.** See Appendix C. ■

The first-best unbundling price is decreasing in  $\beta$  and increasing in  $\alpha$ , as when cost reduction is relatively less expensive, the regulator wants the incumbent to invest relatively more in this type of investment than in quality upgrades. Therefore, he sets a lower unbundling price. However, when it hits the non-negativity condition in the incumbent's profit, the unbundling price set is the second-best one, and this is increasing in both  $\beta$  and  $\alpha$ , as when an investment becomes more expensive, the restriction in incumbent's profit becomes tighter, and therefore the unbundling price must be higher.

De Bijl and Peitz (2004) argue that the regulator can give stronger incentives for an incumbent to invest in the quality of its network by increasing the sensitivity of the unbundling price to the quality level. In fact, if the regulator could set an unbundling price dependent on investment, it should increase in both investment types. In this case, the incumbent would have a higher incentive to invest both in quality and cost reduction, in order to receive a higher unbundling price.

### 5.3 No commitment to unbundling price before investment

In the previous section we assumed that the regulator acts as a first-mover and sets the unbundling price before the incumbent invests. This commitment may, however, not be credible if the regulator can change price at will later on. In this case, he has all the incentives to revise his decision after observing the investment made by the incumbent. Knowing this, the incumbent takes into account how the regulator will change his decision about unbundling price when it invests.

We start by solving the regulator's problem. Given the value of  $g$  and  $h$  chosen by the incumbent in the first stage, the regulator maximizes social welfare with respect to  $r$ .

**Proposition 8** *When the regulator sets the unbundling price after the investment decision, it is such that the incumbent's profit is equal to minus its investment costs.*

**Proof.** See Appendix B, but instead of  $x_0$  and  $y_0$  consider  $x_0 + g + h$  and  $y_0 + h$ . ■

**Corollary 9** *The incumbent does not invest in its network if the regulator only sets the unbundling price ex post. Therefore, welfare is lower as compared with the commitment context.*

**Proof.** See Appendix D. ■

When the regulator acts as a second-mover, he only cares for low equilibrium prices, which are favoured by a low unbundling price. The incumbent foresees this behavior by the regulator, and thus it does not invest. In fact, every gain from its investment is expropriated later by a low unbundling price, and as when the regulator sets the final unbundling price the investment cost is already a sunk cost, the incumbent ends up earning negative

profits if it invests. Note that, in this case, we cannot observe any relationship between the unbundling price and investment. What we observe is that under unbundling there is no investment.

If a regulator cannot commit to his decisions, unbundling policies affect welfare negatively. This result supports the criticisms of service-based competition, namely of its impact on dynamic efficiency. Indeed, if a regulator is implementing an unbundling policy he must show to the market participants that he has the ability to commit to his decisions. If he cannot commit, it may be better to leave the market unregulated as we will see next.

## 6 Unregulated market

When there is no regulator in the market, the incumbent takes all decisions regarding investment and unbundling price. In this case, a high  $r$  is equivalent to no unbundling.

As we have a simultaneous decision over  $(r, g, h)$ , by the envelope theorem, we just need to substitute the optimal investment functions (20) and (21) into the incumbent's profit function (14), and then maximize it with respect to  $r$ . The profit-maximizing *ex ante* access margin becomes:

$$y_0^{ur} = (2 + \theta) \frac{(1 + \theta) (4 - 2\theta + \theta^2) \beta - (6 - \theta + \theta^2)}{T(\theta, \alpha, \beta)} \alpha x_0, \quad (26)$$

where

$$T(\theta, \alpha, \beta) = 2(1 + \theta) (8 + \theta^2) \beta \alpha - (2 + \theta) (6 - \theta + \theta^2) (\beta + \alpha), \quad (27)$$

which is positive for  $\beta > \underline{\beta}^{ur} \equiv \frac{(2+\theta)(6-\theta+\theta^2)\alpha}{2(1+\theta)(8+\theta^2)\alpha - (2+\theta)(6-\theta+\theta^2)}$ .<sup>10</sup>

Therefore, profit-maximizing investments are:

$$g^{ur} = \frac{(2 + \theta) (6 - \theta + \theta^2) \beta}{T(\theta, \alpha, \beta)} x_0 \quad (28)$$

$$h^{ur} = \frac{(2 + \theta) (6 - \theta + \theta^2) \alpha}{T(\theta, \alpha, \beta)} x_0. \quad (29)$$

**Proposition 10** *The incumbent prefers to rent out its loops to remaining on as a monopolist.*

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<sup>10</sup>This condition is also necessary for a negatively defined Hessian.

**Proof.** See Appendix E. ■

The incumbent unbundles its access network in order to attract more consumers to the market. Then, it sets a high unbundling price to absorb part of the profit the entrant earns with these new subscribers, increasing its own profit.

**Proposition 11** *The profit-maximizing unbundling price is higher than the socially optimal one in a commitment context. Therefore, for  $\beta < \tilde{\beta}(\theta)$  the incumbent invests less in both types of investment in an unregulated market, while for  $\beta > \tilde{\beta}(\theta)$  it invests more in quality upgrades and less in cost reduction. Social welfare is always lower as compared to a commitment context.*

**Proof.** We have  $y_0^{ur} > y_0^c$ . The second part follows from Propositions 3 and 4. For the welfare result, we just need to see that the regulator could always have chosen  $y_0^{ur}$  when he set the socially optimal unbundling price. ■

**Proposition 12** *Define  $\beta^*(\theta, \alpha)$  by  $W^{nc} = W^{ur}$ , where  $W^{nc}$  is welfare in a no-commitment context and  $W^{ur}$  is welfare in an unregulated context. The profit-maximizing unbundling price is higher than the socially optimal one in a no-commitment context, and the incumbent invests more in both types of investment. Social welfare is lower as compared to a no-commitment context if  $\beta > \beta^*(\theta, \alpha)$ , and higher if  $\beta < \beta^*(\theta, \alpha)$ .*

**Proof.** See Appendix F. ■

We find that if the regulator can commit to his decisions, social welfare is higher when he intervenes in the market *ex ante* as compared to no intervention since the unbundling price the incumbent would set is too high as compared to the one set by the regulator. Hence, in this case, it is better to have the regulator intervening.

On the other hand, if we compare the unregulated market with the context where the regulator cannot commit, the profit-maximizing unbundling price is still higher, but now welfare can be lower or higher depending on the level of investment costs. Indeed, when investment costs are low, it is preferable to leave the market unregulated since the incumbent unbundles and invests, while with *ex post* regulation we obtain a zero investment by the incumbent. For  $\beta > \beta^*(\theta, \alpha)$  the investment objectives become less important as their socially optimal values are low, and thus regulation *ex post* becomes preferable since it assures low subscription prices.

## 7 Mature markets

In mature markets, we usually have full-consumer participation, which means that total demand is perfectly inelastic with respect to price changes. A relevant example is fixed telephony.

De Bijl and Peitz (2004) show that, contrary to the context of partial consumer participation, an increase in the unbundling price is totally passed on by the entrant to consumers. Bourreau and Dogan (2005) show that this is only true when the marginal consumer obtains positive surplus, which happens for low values of  $r$ .

Bourreau and Dogan (2005) use a Hotelling model to formalize a mature market, where, in equilibrium, each operator is located at one of the extremes of the line. The incumbent's profit and social welfare are given by:

$$\pi_I = \begin{cases} \frac{1}{2} + r - c & \text{if } r \in \left[0, v - \frac{5}{4}\right) \\ \frac{1}{2} \left(v - \frac{1}{4} + r\right) - c & \text{if } r \in \left[v - \frac{5}{4}, v - \frac{3}{4}\right) \\ r - 1 + \sqrt{3}\sqrt{v-r} - \frac{2\sqrt{3}}{9}(v-r)^{\frac{3}{2}} - c & \text{if } r \in \left[v - \frac{3}{4}, v\right) \end{cases}, \quad (30)$$

and

$$W = \begin{cases} v - \frac{1}{12} - c & \text{if } r \in \left[0, v - \frac{3}{4}\right) \\ \frac{2}{3}v + \frac{1}{3}r - \frac{1}{3} - c + \frac{\sqrt{3}}{3}\sqrt{v-r} & \text{if } r \in \left[v - \frac{3}{4}, v\right) \end{cases}, \quad (31)$$

where  $v > 3$  is the fixed utility of consumption

Let us first consider an investment in quality. As in previous sections this investment increases the reservation price, i.e., it increases  $v$ . In this case, when the unbundling price is low ( $r < v - \frac{5}{4}$ ) the incumbent has no incentive to invest since the increases in quality are totally passed on to consumers. For intermediate values of  $r$  the incumbent invests in quality upgrades because this increases the valuation of the marginal consumer, allowing firms to increase prices. For high values of  $r$  the incumbent invests even more, and the investment is increasing in  $r$ . In fact, as for  $r > v - \frac{3}{4}$  the entrant's market share increases when the incumbent invests, the higher is  $r$  the higher is the incentive to invest.

As concerns cost reduction, the incumbent always invests, but the size of the investment is independent of the unbundling price. This is because the number of subscribers is fixed, and translates into the lack of interaction between  $r$  and  $c$  in  $\pi_I$ .

We can conclude that for a full-consumer participation model total investment depends positively on the unbundling price for a sufficiently high  $r$ , and therefore the socially optimal unbundling price must be high enough to give incentives for the incumbent's investment.

## 8 Conclusions

The main objective of this paper is trying to bring some light into the contradictory results in the empirical literature about the effects of unbundling on incumbent firms' investment. Hence, we develop a model with two telecommunications operators, an incumbent and an entrant, that offer differentiated services in a market with partial consumer participation. The incumbent can invest in quality upgrades and in cost reduction, which are complements but have different impacts on both firms. We conclude that both empirically observed relationships are possible. In fact, a low unbundling price increases the intensity of competition, which gives incentives for an incumbent to invest in cost reduction in order to gain a cost advantage for a given unbundling price. On the other hand, it decreases the incumbent's return from investing in quality upgrades. Thus, although one should expect to have both investments moving together due to their complementarity, it is not obvious what the equilibrium effect of a lower unbundling price will be.

Secondly, we compare social welfare when the regulator can commit to an unbundling price set *ex ante* and when he cannot. We show that in the latter case the incumbent does not invest since it does not retain any gain from its investments. As a consequence, social welfare is lower. Here, it may be better to let the market unregulated since the incumbent firm will unbundle its local loop and invest. Thus, for the welfare effects of unbundling policies it is decisive whether the regulator can or cannot commit to unbundling prices.

### Appendix A

We start to solve price competition in the absence of investment.

The representative consumer maximizes the utility function:

$$U = a(q_I + q_E) - \frac{1}{2}(q_I^2 + 2\theta q_I q_E + q_E^2) - p_I q_I - p_E q_E.$$

First-order conditions are:

$$p_I = a - q_I - \theta q_E \tag{32}$$

$$p_E = a - \theta q_E - q_I. \tag{33}$$

Inverting equations (32) and (33), we obtain:

$$q_I = \frac{a(1 - \theta) - p_I + \theta p_E}{1 - \theta^2} \tag{34}$$

$$q_E = \frac{a(1 - \theta) - p_E + \theta p_I}{1 - \theta^2}. \tag{35}$$

Given (34) and (35), the incumbent and entrant's profit function become:

$$\pi_I = \left( \frac{a(1-\theta) - p_I + \theta p_E}{1-\theta^2} \right) (p_I - c) + \left( \frac{a(1-\theta) - p_E + \theta p_I}{1-\theta^2} \right) (r - c)$$

$$\pi_E = (p_E - r) \left( \frac{a(1-\theta) - p_E + \theta p_I}{1-\theta^2} \right).$$

If we maximize each profit function with respect to the price of the respective operator, we obtain the following best response functions:

$$p_I = \frac{1}{2}(a+c)(1-\theta) + \frac{1}{2}\theta p_E + \frac{1}{2}\theta r$$

$$p_E = \frac{1}{2}a(1-\theta) + \frac{1}{2}\theta p_I + \frac{1}{2}r.$$

Solving these, equilibrium prices become:

$$p_I = \frac{1-\theta}{2-\theta}(a-c) + \frac{3\theta}{4-\theta^2}(r-c) + c$$

$$p_E = \frac{1-\theta}{2-\theta}(a-c) - 2\frac{1-\theta^2}{4-\theta^2}(r-c) + r.$$

Substituting both in (34) and (35) we find:

$$q_I = \frac{1}{2-\theta^2+\theta}(a-c) - \frac{\theta}{4-\theta^2}(r-c)$$

$$q_E = \frac{1}{2-\theta^2+\theta}(a-c) - \frac{2}{4-\theta^2}(r-c).$$

Therefore, equilibrium profits and welfare are:

$$\pi_I = \frac{1-\theta}{(1+\theta)(2-\theta)^2}(a-c)^2 - \frac{8+\theta^2}{(4-\theta^2)^2}(r-c)^2$$

$$+ \frac{4-2\theta+\theta^2}{(2-\theta)(4-\theta^2)}(a-c)(r-c)$$

$$\pi_E = \frac{1-\theta}{(1+\theta)(2-\theta)^2}(a-c)^2 + \frac{4(1-\theta^2)}{(2+\theta)^2(2-\theta)^2}(r-c)^2$$

$$- \frac{4(1-\theta)}{(2-\theta)^2(2+\theta)}(a-c)(r-c)$$

$$W = \frac{3 - 2\theta}{(2 + \theta - \theta^2)(2 - \theta)} (a - c)^2 - \frac{4 + 5\theta^2}{2(4 - \theta^2)^2} (r - c)^2 - \frac{1 - \theta}{(2 - \theta)^2} (a - c)(r - c).$$

Finally, we introduce the two investments into the equilibrium and obtain  $a + g$  instead of  $a$ , and  $c - h$  instead of  $c$ . We also have to introduce the investment cost functions (8) and (9) into welfare and incumbent's profit.

### Appendix B. Proof of Proposition 6

The regulator maximizes social welfare subject to the non negativity condition in the incumbent's profit. Thus, we have the following Lagrangian:

$$\mathcal{L} = W(y, x, \theta) + \lambda_1 \pi_I.$$

First-order conditions are:

$$-2 \frac{(1 - \theta)(2 + \theta)^2 x_0 + (4 + 5\theta^2) y_0}{(4 - \theta^2)^2} + \lambda_1 \frac{(8 + \theta^3) x_0 - 2y_0(8 + \theta^2)}{(4 - \theta^2)^2} = 0 \quad (36)$$

$$\pi_I \geq 0, \quad \lambda_1 \geq 0, \quad \pi_I \lambda_1 = 0.$$

If  $\lambda_1 = 0$ , we obtain:

$$y_0 = -\frac{(1 - \theta)(2 + \theta)^2}{4 + 5\theta^2} x_0,$$

but this violates  $\pi_I \geq 0$  restriction.

If  $\lambda_1 > 0$ , we obtain  $\pi_I = 0$ , and thus (36) becomes:

$$\lambda_1 = 2 \frac{x_0(1 - \theta)(2 + \theta)^2 + (4 + 5\theta^2) y_0}{(8 + \theta^3) x_0 - 2y_0(8 + \theta^2)} > 0.$$

Therefore, the second-best socially optimal access margin is

$$y_0^{sb} = (2 + \theta) \frac{(1 + \theta)(4 - 2\theta + \theta^2) - (2 - \theta) \sqrt{(2 + \theta)(1 + \theta)(6 - \theta + \theta^2)}}{2(1 + \theta)(8 + \theta^2)} x_0. \quad (37)$$

This access margin is lower than  $\bar{y}_0$ , and thus the entrant asks for access.

### Appendix C. Proof of Proposition 7

First, we substitute the profit-maximizing investments (20) and (21) into the welfare function (16) and then maximize with respect to  $y_0$  to obtain the first-best access margin  $y_0^{c1} = Z(\theta, \alpha, \beta) x_0$ .

Second, we define the incumbent's profit at the profit-maximizing investments by

$$\pi_I^*(\theta, \alpha, \beta, x_0, y_0) = \pi_I(\theta, \alpha, \beta, x_0, y_0, g^*(\theta, \alpha, \beta, x_0, y_0), h^*(\theta, \alpha, \beta, x_0, y_0)).$$

Then we introduce the first-best access margin and find that:

$$\pi_I^*(\theta, \alpha, \beta, x_0, y_0^{c1}(\theta, \alpha, \beta, x_0)) \geq 0,$$

if and only if,  $\beta \geq \bar{\beta}(\theta, \alpha)$  (this is independent of  $x_0$ ). Thus, for  $\beta \geq \bar{\beta}(\theta, \alpha)$ , the socially optimal *ex ante* access margin is  $y_0^{c1} = Z(\theta, \alpha, \beta) x_0$ , and we have:

$$\frac{\partial Z(\theta, \alpha, \beta)}{\partial \alpha} < 0, \frac{\partial Z(\theta, \alpha, \beta)}{\partial \beta} > 0.$$

When  $\beta < \bar{\beta}(\theta, \alpha)$ , we have

$$\pi_I^*(\theta, \alpha, \beta, x_0, y_0^{c1}(\theta, \alpha, \beta, x_0)) < 0,$$

and therefore, the second-best socially optimal *ex ante* access margin is obtained by solving  $\pi_I^*(\theta, \alpha, \beta, x_0, y_0^{c2}) = 0$ , which gives  $y_0^{c2} = U(\theta, \alpha, \beta) x_0$  where:

$$\frac{\partial U(\theta, \alpha, \beta)}{\partial \alpha} > 0, \frac{\partial U(\theta, \alpha, \beta)}{\partial \beta} > 0.$$

The socially optimal *ex ante* access margin under commitment is then  $y_0^c = \max\{y_0^{c1}, y_0^{c2}\}$  and, after tedious calculations, it is possible to prove that  $y_0^c < \bar{y}_0$ .

#### Appendix D. Proof of Proposition 9

The first part of the Proposition is a natural consequence of Proposition 8, since if the incumbent invests it will earn negative profits. The *ex ante* access margin set by the regulator is then given by (37), and social welfare becomes:

$$W^{nc} = \left( \frac{(128 + 16\theta + 22\theta^2 - 4\theta^3 + 3\theta^4 - 3\theta^5)}{4(1 + \theta)(8 + \theta^2)^2} - \frac{(24 + 6\theta^2 - 3\theta^3) \sqrt{(2 + \theta)(1 + \theta)(6 - \theta + \theta^2)}}{4(1 + \theta)(8 + \theta^2)^2} \right) x_0^2.$$

Comparing  $W^{nc}$  with welfare in a commitment context ( $W^c$ ) it is possible to show, after tedious calculations, that  $W^c > W^{nc}$ .

### Appendix E. Proof of Proposition 10

$y_0^{ur}$  is such that the entrant asks for access since  $y_0^{ur} < \bar{y}$ . Thus, we can introduce  $y^{ur}$ ,  $g^{ur}$  and  $h^{ur}$  into the incumbent's profit function (14) and obtain:

$$\pi_I^{ur} = \frac{(2 + \theta)(6 - \theta + \theta^2)\alpha\beta}{2T(\theta, \alpha, \beta)}x_0^2.$$

A monopolist incumbent solves the following problem:

$$\max_{g, h} \frac{1}{4}(x_0 + h + g)^2 - \frac{\alpha}{2}g^2 - \frac{\beta}{2}h^2,$$

which gives optimal investments under monopoly:

$$g^M = \frac{\beta}{2\alpha\beta - \alpha - \beta}x_0$$

$$h^M = \frac{\alpha}{2\alpha\beta - \alpha - \beta}x_0.$$

Thus, incumbent's monopoly profit is:

$$\pi_I^M = \frac{1}{2} \left( \frac{\alpha\beta}{2\alpha\beta - \alpha - \beta} \right) x_0^2.$$

If we compare with profit under unbundling, we find that  $\pi_I^{ur} \geq \pi_I^M$ .

### Appendix F. Proof of Proposition 12

If we substitute (26), (28), (29) into (16), we obtain welfare in an unregulated market:

$$W^{ur} = \left( \frac{(1 + \theta)(304 + 48\theta + 108\theta^2 + 16\theta^3 + 11\theta^4 - \theta^5)\beta\alpha}{T(\theta, \alpha, \beta)^2} - \frac{(\beta + \alpha)(2 + \theta)^2(6 - \theta + \theta^2)^2}{T(\theta, \alpha, \beta)^2} \right) \frac{1}{2}\alpha\beta x_0^2.$$

Comparing with  $W^{nc}$  we obtain that  $W^{nc} > W^{ur}$  for  $\beta > \beta^*(\theta, \alpha)$ .

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